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HALL EFFECTS ON ANOMALOUS HEAT, PARTICLE AND HELICITY TRANSPORTS THROUGH TEARING-MODE TURBULENCE

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Hall Effects on Anomalous Heat, Particle and Helicity Transports through Tearing-Mode Turbulence

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ABSTRACT

The helicity transport in a current-carrying plasma results in heat and particle transports in the direction opposite to the helicity flux. Tearing-mode turbulence produces helicity flux that is proportional to the gradient of the equilibrium parallel current. The helicity flux is a consequence of a fluctuating electric field with a circularly polarized component, which also causes a nonlinear parallel current (primarily an electron flux) and a nonlinear polarization current (primarily an ion flux). Such anomalous heat and particle fluxes are driven by the free-energy associated with the perturbed magnetic field in the tearing-mode turbulence, and are typically directed inward to the plasma. Both fluxes becomes large when the gradient of the equilibrium current is large.

I. INTRODUCTION

In a toroidal discharge, a large fraction of the plasma current is directed parallel to the equilibrium magnetic field, i.e. force free. Such a parallel current is primarily driven by a toroidal electric field applied by external induction. On the other hand, the perpendicular component of the current, for example the diamagnetic current related to a pressure gradient, should be sustained, against the resistive dissipation, by an internal electric field that is expressed by $\mathbf{v}\times\mathbf{B}$ in the fluid-dynamic model, where $\mathbf{v}$ is the fluid-dynamic velocity and $\mathbf{B}$ is the magnetic flux density. With taking $\mathbf{B} = \mathbf{B}_0$ (the mean field), we obtain a cross-field particle flux that leads to the classical estimate of the transport (B^{-2} formula).[1,2]

Let us re-examine the parallel component with respect to $\mathbf{B}_0$: with time-averaging the equation of electrons we obtain

$$\eta j_{//,0} - \mathbf{E}_D\cdot\mathbf{b}_0 = \left\langle \mathbf{v}_{H,1}\times\mathbf{B}_1\cdot\mathbf{b}_0\right\rangle + e^{-1}\left\langle n_{e,1}^{-1}\nabla p_{e,1}\right\rangle, \tag{1.1}$$

where subscripts 0 and 1, respectively, represent the mean value and perturbation, $j_{//}$ is the parallel current, η is the parallel resistivity, $\mathbf{b}_0 = \mathbf{B}_0/B_0$, $\mathbf{E}_D$ is the driving electric field, $\mathbf{v}_H = \mathbf{v} - \mathbf{j}_{//}/en_e$, n_e is the electron density, e is the elementary charge and p_e is the electron pressure. Here we neglect the electron inertia term. In low-frequency fluctuations, the Hall term and the last term of the right-hand side of (1.1) can be neglected. The left-hand side of (1.1) consists of classical terms, while the right-hand-side terms are the correlations of fluctuations. In the classical limit, these fluctuation terms are assumed to vanish. Then we obtain $\eta j_{//,0}$ being approximately constant in a steady state. There are, however, a plenty of experimental observations of anomalous transports of $j_{//}$ in quasi-static discharges, where the fluctuation terms are necessary to account for the electric-field balance on magnetic field lines.

The first term in the right-hand side of (1.1) represents the so-called dynamo effect. We see that a finite $\mathbf{B}_1$ that is perpendicular to $\mathbf{B}_0$ is required for the dynamo term to be

finite. We thus may expect enhanced transports of heat and particles related to $\mathbf{B}_1$, when there is an anomalous transport of $j_{//}$. A well-known effect is the parallel electron heat conduction through stochastic field lines.[3] The electron heat flux by thermal conduction is given by

$$q_B^e = - \chi_B^e \, n_{e,0} \nabla T_{e,0}, \tag{1.2}$$

where T_e is the electron temperature,

$$\chi_B^e = v_{e,th} \, L_\tau \left\langle \left(\frac{B_{x,1}}{B_0} \right)^2 \right\rangle, \tag{1.3}$$

$v_{e,th}$ is the electron thermal velocity, L_τ is the correlation length and $B_{x,1}$ is the radial component of the perturbed magnetic field.

There is another mechanism of transports that is related to $\mathbf{B}_1$ in a different way. This mechanism of heat and particle transports is not based on conductions, but is caused in response to the free energy associated with the transport of $j_{//}$. The heat and particle fluxes as results of such transport mechanism can be directed inward to the plasma depending on the sign of the gradient of $j_{//,0}$. The transport of $j_{//}$ is formulated by using the helicity. The related heat and particle transports are shown to appear if and only if a finite helicity flux is caused by the perturbed electromagnetic fields. In the next section, we briefly review the physics of the helicity transport.

II. BACKGROUND OF HELICITY TRANSPORT

The helicity of magnetic field describes structures of the parallel current and the corresponding force-free component of the magnetic field in a plasma. The well known helicity balance equation is

$$\frac{d}{dt}\int_{\Omega} \mathbf{A}\cdot\mathbf{B}\ dv = -\int_{\partial\Omega} \mathbf{n}\cdot\left[\left(-\partial_t\mathbf{A}\right)\times\mathbf{A} + 2\mathbf{B}\varphi\right]ds - 2\int_{\Omega} \eta\mathbf{j}\cdot\mathbf{B}\ dv ,$$

where Ω is a fixed volume, $\nabla\times\mathbf{A}=\mathbf{B}$ and φ is the electrostatic potential. The gauge should be fixed throughout the evolution. The first term of the right-hand side represents the helicity flux, while the second one, the helicity dissipation because of the parallel resistivity. Since the dissipation of the helicity is caused by the parallel current, the helicity is a quantity that is primarily related to parallel currents.[4] We write perturbation

$$\mathbf{A}_1 = \nabla u \times \nabla z + \psi\nabla z ,$$

which yields

$$\mathbf{B}_1 = \nabla\psi \times \nabla z - \left(\Delta u\right)\nabla z .$$

The wave-number vector $\mathbf{k}$ is assumed to be in y-z planes (local mode approximation), and we take $\mathbf{k} = k\nabla y$. When we take a specific gauge such that $\varphi = 0$ at $x = 0$, the helicity flux across the surface $x = 0$ is calculated as

$$\langle F_h\rangle_x \equiv \left\langle -(\partial_t\mathbf{A}_1)\times\mathbf{A}_1\cdot\nabla x\right\rangle = \left\langle \psi(\partial_{t,x}u) - (\partial_t\psi)(\partial_x u)\right\rangle . \tag{2.1}$$

This relation immediately shows that, for the helicity flux to be finite, magnetic fields must be perturbed. When we neglect the displacement current with assuming low-frequency perturbations, we can easily calculate the corresponding current density in the plasma. We obtain the nonlinear parallel current

$$\langle j_{//}\rangle_x \equiv \left\langle (\mathbf{j}\cdot\mathbf{b})\mathbf{b}\cdot\nabla x\right\rangle$$

$$= \frac{\left(\mathbf{b}_0\cdot\nabla y\right)}{\mu_0 B_0}\left\langle(\partial_x\Delta u)(\partial_y\psi)\right\rangle - \frac{\left(\mathbf{j}_0\cdot\nabla z\right)}{B_0^2}\left\langle(\Delta u)(\partial_y\psi)\right\rangle$$

$$= \frac{1}{\mu_0 B_0}\left\langle(\partial_x \Delta u)(ik_{//}\psi)\right\rangle - \frac{j_{//,0}}{B_0^2}\left\langle(\Delta u)(ik_{\perp}\psi)\right\rangle \quad , \tag{2.2}$$

where **b** = **B**/B that includes the perturbation, subscript 0 impels the mean value, $k_{//} = \mathbf{k}\cdot\mathbf{b}_0$, and $k_{\perp} = k(\mathbf{b}_0 \cdot \nabla z)$.

The first term of $\langle j_{//}\rangle_x$ in (2.2) becomes finite, when ensemble average of $k_{//}$ is finite. Since the sign of $k_{//}$ represents the direction of the Poynting vector with respect to the mean field, this sign is essential to characterize the anisotropy of the perturbations that transports the helicity. On the other hand, the second term of $\langle j_{//}\rangle_x$, which is proportional to $k_{\perp}$, vanishes as far as the perturbation is isotropic with respect to $k_{\perp}$, and the relation between the perturbations u and ψ is independent of the sign of $k_{\perp}$. The latter condition turns out to be true when gradients in mean quantities are negligible; see Sec. IV. Under these assumptions, we neglect the second term in (2.2) to obtain[5]

$$\langle j_{//}\rangle_x = \frac{-(k_{//}/\omega)k^2}{2\mu_0 B_0}\langle F_h\rangle_x \quad . \tag{2.3}$$

Here the wave quantities are considered to be ensemble means over the fluctuations. The parallel current is primarily carried by electrons in normal discharge plasmas. This relation shows that a finite helicity flux through a mean-field magnetic surface is accompanied by a finite flux of electrons, as far as the relevant fluctuations have a coherence to cause an average phase velocity.

Equation (2.3) immediately relates the parallel frictional component q^e_u of the electron heat flux[6] with the helicity flux

$$\langle q^e_u\rangle_x \equiv \left\langle q^e_u \cdot \nabla x\right\rangle = C_z T_e n_e \langle j_{//}\rangle_x / (-e n_e)$$

$$= \frac{C_z T_e (k_{//}/\omega)k^2}{2\mu_0 B_0 e}\langle F_h\rangle_x \quad , \tag{2.4}$$

where C_Z is a positive constant of order 1 ($C_Z = 0.71$, if $Z=1$). Since this heat flux is caused by a divergence in the parallel current, the average charge neutrality condition requires a balanced perpendicular current. In the low-frequency ($\omega << \omega_{ci}$) regime, the nonlinear polarization drift of the ions dominates the perturbed perpendicular current;

$$\langle j_P \rangle_x = \frac{\rho_m}{B_0^2} \left\langle (\mathbf{v}_1 \cdot \nabla) \mathbf{E}_1 \right\rangle \cdot \nabla x$$

$$= -\frac{\rho_m (\mathbf{b}_0 \cdot \nabla y)}{B_0^3} \left\langle (\partial_{t,x,y} u)(\partial_t \psi) + (\partial_x^2 \varphi_1)(\partial_t \psi) \right\rangle \quad , \qquad (2.5)$$

where ρ_m is the mass density of the ions, the perturbed velocity $\mathbf{v}_1$ is dominated by the $\mathbf{E}\times\mathbf{B}$ drift, the radial electro-static field is determined by the average charge neutrality condition, which reads

$$\partial_x^2 \varphi_1 = \left[1 - (v_A k/\omega)^2 \right] \partial_{t,x,y} u \, ,$$

where v_A is the Alfvén velocity. The electron flux by the nonlinear parallel current, and the ion flux by the nonlinear polarization drift result in particle flux

$$\langle \Gamma \rangle_x = -\langle j_{//} \rangle_x / e = \langle j_P \rangle_x / e$$

$$= \frac{(k_{//}/\omega) k^2}{2 \mu_0 B_0 e} \langle F_h \rangle_x \quad . \qquad (2.6)$$

We note that all three correlations, the helicity flux, nonlinear parallel current, and the nonlinear polarization current can be finite to cause the heat and particle fluxes, when the perturbed electric field satisfies the same phase and polarization relation. The heat and particle fluxes given by (2.4) and (2.6) are proportional to the helicity flux, and are independent to the gradients in the temperature or the density. A significant amount of the heat and particle

fluxes are shown to be caused, for example when a circularly polarized Alfvén wave propagates across magnetic surfaces to inject the helicity.[5]

When there is a gradient of the equilibrium parallel current density, a different term arising from instabilities with $k_{//} \approx 0$ contributes to $<j_{//}>_X$. The second term of $<j_{//}>_X$ in the right-hand side of (2.2),

$$- j_{//,0} B_0^{-2} \left\langle (\Delta u), ik_\perp \psi \right\rangle,$$

may survive even if the turbulence is isotropic with respect to $k_\perp$, when we consider the Hall effect and a gradient of $j_{//,0}$. The electron heat flux turns out to be directed in the direction opposite to the anomalous diffusion of $j_{//,0}$. This transport process is closely related to the helicity transport caused by the turbulences of the tearing modes. It is known that the quasilinear tearing-mode turbulence results in the hyper-resistivity, which is equivalent to the divergence of the helicity flux caused by turbulent reconnections. In the next sections, we shall calculate the relation among the helicity flux, electron heat flux, and the particle flux caused by the tearing-mode turbulence.

III. HELICITY TRANSPORT IN TEARING-MODE TURBULENCE

The following calculations are based on the tokamak reduced MHD equations[7] with including the two-fluid effect by the Hall term. We write

$$\mathbf{B} = \nabla\psi\times\nabla z + B_z\nabla z ,$$

$$\mathbf{v}_H = \nabla\phi\times\nabla z + v_z\nabla z ,$$

with assuming $B_{z,0}$ = constant (low β), $\psi_0 = \psi_0(x)$, $\nabla\psi_0 = 0$ at x=0, $\phi_0 = 0$, and $v_{z,0} = - j_{//,0}/(en_e) = v_{z,0}(x)$ which is the mean flow velocity of the electrons. The linear equations of generalized Ohm's law reads

$$\mathbf{E} = \eta\mathbf{j} - \mathbf{v}_H \times \mathbf{B}$$

$$= \eta \mathbf{j} - \left[\left(\nabla\phi_1 \times \nabla\psi_0 \right) \cdot \nabla z + B_{z,0}\partial_z\phi_1 - v_{z,0}\partial_z\psi_1 \right] \nabla z$$
$$+ B_{z,0}\nabla\phi_1 - v_{z,0}\nabla\psi_1 - v_{z,1}\nabla\psi_0 \qquad (3.1)$$

Here, we assume a low-β plasma and neglect the electron pressure terms. Otherwise, there appear additional terms, related to perturbations in T_e and n_e, in the following calculations of various correlations.[8] We consider resonant-mode instabilities driven by gradients in $j_{//,0}$, i.e. tearing modes. The tearing-mode turbulence behaves to flatten $j_{//,0}$. This effect is clearly formulated by the hyper-resistivity term deduced by the quasilinear theory.[9-11] We can show that the hyper-resistivity term is equivalent to the helicity-flux term in the helicity balance equation.

The helicity flux is given by

$$\langle F_h \rangle_x = \langle E_y \psi \rangle_x - \langle E_z A_y \rangle_x + \left\langle \varphi \partial_y \psi \right\rangle . \qquad (3.2)$$

A Fourier component of the first term of $<F_h>_x$ may be expressed as

$$E_{y,k} \cdot \psi_k^* = B_{z,0}\partial_y\phi_k \cdot \psi_k^* = -\frac{B_{z,0}^2}{\mu_0 j_{//,0}} \frac{\gamma}{x_k} |\psi_k|^2 , \qquad (3.3)$$

where subscript k represents an unstable mode such that $k_{//}(x_k) = 0$, γ is the growth rate of the instability. Here we use the resonance condition $\mathbf{k} \approx k_\perp \nabla y$, which yields

$$\frac{d\,k_{//}}{d\,x} \approx k_\perp \frac{d\,b_{0,y}}{d\,x} = k_\perp \mu_0 j_{//,0} / B_{z,0} ,$$

and an ideal linear approximation of the induction equation

$$\gamma\psi_k = i k_{//}\phi_k$$

to write

$$\phi_k = \frac{-\gamma B_{z,0}\psi_k}{ikx_k\mu_0 j_{//,0}} . \tag{3.4}$$

Neglecting ηj_k in (3.1) and using (3.4), we obtain (3.3).

For resonant modes, the second term of $\langle F_h \rangle_x$ is negligible. In fact, using (3.4), we have

$$E_{z,k} \approx -ik\phi_k(\partial_x\psi_0) \approx \gamma B_{z,0}\psi_k(\partial_x\psi_0)/(x_k j_{//,0}) . \tag{3.5}$$

Neglecting ηj_k in (3.1), we obtain the perturbed parallel magnetic field

$$\begin{aligned} \partial_x A_{y,k} - ikA_{x,k} &= B_{z,k} \\ &= -\gamma^{-1}(\nabla\times\mathbf{E}_k)\cdot\nabla z = \gamma^{-1}(\partial_x v_{z,0})(\partial_y\psi_k) . \end{aligned} \tag{3.6}$$

Using the Coulomb gauge condition and (3.6), we obtain

$$A_{y,k} = \Delta^{-1}\,\partial_x\left[\gamma^{-1}ik\psi_k(\partial_x v_{z,0})\right] ,$$

which, together with (3.5), shows that the correlation of the second term vanishes.

The third term in the right-hand side of (3.2) of $\langle F_h \rangle_x$ includes the electrostatic potential φ. By taking the divergence of (3.1), we obtain

$$\begin{aligned} -\Delta\varphi_k &= \nabla\cdot\mathbf{E}_k \\ &= B_{z,0}\Delta\phi_k - v_{z,0}\Delta\psi_k - \nabla v_{z,0}\cdot\nabla\psi_k - \nabla v_{z,k}\cdot\nabla\psi_0 - v_{z,k}\Delta\psi_0 . \end{aligned}$$

Since $-\Delta\psi_k = \mu_0 j_{z,k} = -en_e v_{z,k}$, ψ_k and $v_{z,k}$ are in-phase. We thus obtain

$$\varphi_k\cdot\partial_y\psi_k^* = -B_{z,0}\phi_k\cdot\partial_y\psi_k^* . \tag{3.7}$$

By (3.3) and (3.7), the helicity flux is given by

$$\langle F_h \rangle_x = -2B_{z,0}\langle\phi_1\cdot B_{x,1}\rangle = -2B_{z,0}\langle\phi_1\cdot\partial_y\psi_1\rangle ,$$

or expressing in Fourier amplitudes,

$$\langle F_{h,k} \rangle_x = -2B_{z,0}\phi_k \cdot \partial_y \psi_k^* = -\frac{2B_{z,0}^2}{\mu_0 j_{//,0} x_k} \gamma |\psi_k|^2 \quad (3.8)$$

Here we note that $\nabla \cdot (\langle F_h \rangle)$ corresponds to the hyper-resistivity term caused by the resonant mode turbulence;

$$\langle F_h \rangle_x = -B_{z,0}\, D_0 \nabla j_{//,0},$$

where D_0 is the hyper-resistivity. This relation is consistent with the previous calculations based on the single-fluid reduced equations.[12] The Hall term acts only to Doppler-shift the turbulence spectra. The quasilinear theory gives an estimate of the hyper-resistivity; we obtain[11]

$$D_0 = \frac{1}{2}\sum_k \frac{\gamma_k}{(\partial_x k_{//})^2} |B_{x,k}|^2 \ln\left(\frac{x_k^2}{(x - x_k)^2 + \gamma_k^2/(\partial_x k_{//})^2}\right).$$

IV. ANOMALOUS TRANSPORTS INDUCED BY HELICITY FLUX

In this section, we calculate the heat and particle fluxes which are correlated with the helicity flux caused by the tearing-mode turbulence. We first estimate the nonlinear parallel current. In the expression (2.2) of $\langle j_{//} \rangle_x$, the second term dominates the nonlinear parallel electron flux for the resonant fluctuations. By using (3.6), we obtain

$$\langle j_{//} \rangle_x = \frac{(j_0 \cdot \nabla z)}{B_0^2} \langle (B_{z,1})(\partial_y \psi_1) \rangle = \frac{j_{//,0}}{B_0^2} (\partial_x v_{z,0}) \langle \gamma^{-1} k_\perp^2 \psi_1^2 \rangle \quad (4.1)$$

This x-component of the nonlinear parallel current appears if the unit vector **b** is perturbed to have a finite radial component $\mathbf{b} \cdot \nabla x$ which stays positive (or negative) longer than

negative (or positive). By (3.8), we can relate this nonlinear parallel current with the helicity flux;

$$\langle j_{//,k}\rangle_x = \frac{-x_k\left(\partial_x v_{z,0}\right)\lambda^2}{\mu_0 B_0^2}\left(\frac{k_\perp}{\gamma}\right)^2\langle F_{h,k}\rangle_x \quad , \tag{4.2}$$

where $\nabla\times\mathbf{B}_0 \approx \lambda\mathbf{B}_0$. When we assume that n_e is spatially constant, we obtain

$$\langle j_{//}\rangle_x \approx \frac{-\partial_x j_{//,0}^2}{2B_0^2 e n_e}\left\langle\gamma^{-1}k_\perp^2\psi_1^2\right\rangle \quad ,$$

which shows that the parallel electron flux $<j_{//}>_x/(-en_e)$ is directed to the gradient of the magnitude of the parallel mean current $j_{//,0}$. The parallel frictional electron heat flux is given by

$$\langle q_u^e\rangle_x = \frac{-C_z T_e}{e}\langle j_{//}\rangle_x = \frac{C_z T_e \partial_x j_{//,0}^2}{2e^2 n_{e,0}}\left\langle\gamma^{-1}\left(\frac{B_{x,1}}{B_0}\right)^2\right\rangle . \tag{4.3}$$

Using (3.8), we may also write the frictional heat flux using $<F_h>_x$;

$$\langle q_{u,k}^e\rangle_x = \frac{C_z T_e}{e}\frac{x_k\left(\partial_x v_{z,0}\right)\lambda^2}{\mu_0 B_0^2}\left(\frac{k_\perp}{\gamma}\right)^2\langle F_{h,k}\rangle_x \quad .$$

Equation (4.2) shows that, for the helicity flux to be finite, the perturbed parallel current should have a finite net flux through the mean-field magnetic surface. We thus require a balances finite average in the perturbed perpendicular current, which is primarily an ion current, to retain the charge neutrality. As a result, a particle flux is caused;

$$\langle\Gamma\rangle_x = \frac{-1}{e}\langle j_{//}\rangle_x = \frac{\partial_x j_{//,0}^2}{2e^2 n_{e,0}}\left\langle\gamma^{-1}\left(\frac{B_{x,1}}{B_0}\right)^2\right\rangle \quad . \tag{4.4}$$

The perpendicular ion current is primarily caused by the nonlinear polarization drift; see (2.5). The polarization current is dependent upon the perturbed radial electrostatic potential. In the expression (3.1) of the electric field, ψ_1, ϕ_1 and $v_{z,1}$ describes the perturbations. Fluxes $\langle F_h \rangle_x$ and $\langle j_{//} \rangle_x$ are independent of $v_{z,1}$, while the polarization current includes $v_{z,1}$, which is to be determined to satisfy the average charge neutrality condition. Using (3.1) and (3.4), we obtain

$$\langle j_p \rangle_x = \frac{\rho_m}{\mu_0 B_0^2 j_{//,0}} \left\langle \gamma x^{-1} \psi_1 \left(\partial_x \psi_1 \cdot \partial_x v_{z,0} + \partial_x^2 \psi_0 \cdot v_{z,1} \right) \right\rangle .$$

The average charge neutrality condition $\langle j_{//} \rangle_x + \langle j_p \rangle_x = 0$ yields

$$v_{z,k} \cdot \partial_x^2 \psi_0 + \left[(\delta_x \lambda)^2 (v_A k_\perp / \gamma)^2 + 1 \right] \partial_x v_{z,0} \cdot \partial_x \psi_k = 0 ,$$

where v_A is the Alfvén velocity, $\partial_x \approx 1/\delta_x$.

V. DISCUSSION

We have derived the heat and particle fluxes (4.3) and (4.4) that are correlated with the helicity flux driven by tearing-mode turbulence. An interesting point is that the both fluxes are normally directed inward to the plasma, since $\nabla j_{//,0}{}^2$ is directed inward. The helicity flux is then outward, which contributes to flatten $j_{//}$. These transports are primarily independent of the temperature or density gradients, and are driven by the free energy associated with the helicity transport. They are different from the stochastic thermal conductions and various anomalous transports based on the correlations among $\mathbf{E}_1 \times \mathbf{B}_0$ drifts and fluctuating density and temperature. In the present mechanism, the electron flux is caused by the nonlinear parallel current, while the ion flux is by the nonlinear polarization current. There two currents are the second order correlations of the perturbation that, as well as the helicity flux, become finite when the polarization and the phase of the perturbed electric

field satisfy the same relation. The tearing-mode turbulence causes a helicity flux, so that it is accompanied by the heat and particle fluxes.

Let us compare the magnitude of the present heat transport with the electron heat flux by the thermal conduction through stochastic field lines; see (1.2). The correlation length L_T in (1.3) is normally of the order of the minor radius of the plasma. Let us assume that both $j_{//,0}$ and $T_{e,0}$ have a same magnitude of characteristic length δ_x of spatial distributions. Here, a finite gradient in $T_{e,0}$ is necessary to compare our result with the stochastic conduction, while the electron pressure gradient should not be large in order to avoid inconsistency with our previous assumption in (3.1). Then we obtain

$$|q^e_{\mathrm{u},x}| / |q^e_{\mathrm{B},x}| \approx v_{z,0}^2 / \left(v_{e,th}\gamma\delta_x\right) = \xi_e v_{z,0} / \left(\gamma\delta_x\right),$$

where ξ_e is the electron streaming factor which is typically of the order of 10^{-3}. In normal tokamak parameters, both $v_{e,th}$ and the Alfvén velocity v_A are of the order of 10^7 m/s. The growth rate γ of the tearing modes are normally much smaller than $1/v_A\delta_x$. When $\gamma/v_A\delta_x$ is of the order of 10^{-3}, then the present heat flux $\mathbf{q}^e{}_u$ makes a significant contribution to the total electron heat transport. An equivalent magnitude of the particle flux is also directed inward to the plasma, and can be large enough to improve the particle confinement if the gradient of the mean current (actually the electron drift velocity) is sufficiently large. The particle loss through stochastic fields is not so large, while the electron thermal conduction behaves as a counter process for the electron heat transports.

For the heat and particle fluxes (4.3) and (4.4), the Hall effect plays an essential role, while, for the helicity flux, it has no explicit influence. Taking the mean velocity of the electrons $v_{z,0} = 0$ is equivalent to the single-fluid limit, where the present heat and particle fluxes vanish. In fact, the single-fluid model predicts a quite small energy flux associated with the helicity flux by the tearing-mode turbulence.[12] Normally the Hall term is considered to have a less influence to the evolution of the electric field and the momentum

(ion motion) of the plasma. It, however, has an essential effect on the transport of the electrons. The perpendicular perturbed current that contributes to the Hall term is primarily an ion current, and the magnitude of the Hall term is of the order of ω/ω_{ci} times the induction term $\mathbf{v}\times\mathbf{B}$. On the other hand, the parallel mean current, which is primarily an electron current, causes a Doppler shift of the fluctuations that is expressed by the first-order Hall term $\mathbf{j}_{//,0}\times\mathbf{B}_1/en_e$. This term should be retained in the linearized electron equation (3.1), because the phase-shift of fluctuations by this effect is important in the calculations of the correlations for the fluxes.

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